1. Ranger the dog is tethered to a pole with an 10 ft long rope at the corner of a building that is shaped like a regular pentagon (when viewed from above). The sides of the building are each 20 ft long, and Ranger is outside the building. How much area, in square feet, can Ranger reach?

   (A) 14π (B) 30π (C) 70π (D) 100π (E) 280π

2. The sequence $0, 1, 1, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 4, ...$ is formed by writing each nonnegative integer n a total of 2^n times, in increasing order. What is the 2007th element of the sequence?

   (A) 9 (B) 10 (C) 11 (D) 512 (E) 1024

3. How many ordered pairs of digits (A, B) are there such that the base ten number $A2007B$ is divisible by 99?

   (A) -1 (B) 0 (C) 1 (D) 2 (E) 3

4. Given that

$$\left(\cdots\left(\left(17^{\frac{2}{3}}\right)^{\frac{3}{4}}\right)^{\frac{4}{5}\cdot^{\cdot^{\cdot}}}\right)^{\frac{99}{100}} = \sqrt[100]{x}$$

   What is x?

   (A) $\sqrt{17}$ (B) $\frac{17}{2}$ (C) 17 (D) 34 (E) 289

5. A caveman is making 5 piles of stones by placing one stone at a time on a randomly chosen pile. What is the smallest number of stones he must use in order to guarantee that some pile contains at least 10 stones?

   (A) 10 (B) 45 (C) 46 (D) 49 (E) 51

6. A spherical container of radius 25 is partially filled with water. The depth (measured perpendicular to the surface) at the deepest part of the water is 18. What is the area of the top surface of the water (the water not touching the sphere)?

   (A) 576π (B) 625π (C) 49π (D) 324π (E) 441π

7. If $\ln 1 + \ln x + \ln x^2 + \cdots + \ln x^{10} = 165$, what is $\ln x$? (Note: $\ln = \log_e$ is the natural logarithm.)

   (A) e^3 (B) e (C) 1 (D) 2 (E) 3

8. Point A has coordinates $(-12, 5)$ and B has coordinates $(12, -5)$. If P is a point in the plane such that $\angle APB$ is a right angle, what is the maximum possible value of the y-coordinate of P?

   (A) 5 (B) 10 (C) 12 (D) 13 (E) 17

9. Ben is sharpening pencils, which are initially cylinders of diameter 1 cm, with a pencil sharpener that can hold 10 cm^3 of pencil shavings. He sharpens each so that the total length of the pencil remains the same, but the tip is a cone with height 2 cm. How many whole pencils can he sharpen before having to empty the sharpener?

   (A) 8 (B) 9 (C) 10 (D) 11 (E) 12

10. Compute $(\sin 22.5° - \cos 22.5°)^2$.

    (A) $1 - \sqrt{2}$ (B) $\frac{1-\sqrt{2}}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{2+\sqrt{2}}{2}$ (E) $\frac{2-\sqrt{2}}{2}$

11. An ellipse has its foci at $(-4, 0)$ and $(4, 0)$, and the sum of the distances of any point on the ellipse to the two foci is 10. There exist exactly two circles centered at the origin that are tangent to the ellipse. Find the sum of the radii of these two circles.

    (A) 4 (B) 5 (C) 7 (D) 8 (E) 9

12. The sum of the squares of three real numbers is 147. Find the maximum possible value of the sum of the three numbers.

    (A) 7 (B) 14 (C) 18 (D) 21 (E) 49

13. In how many different ways can \$100 be made from nickels, dimes, and quarters if exactly 1000 coins must be used?

    (A) 323 (B) 251 (C) 361 (D) 237 (E) 301

14. An octahedron is inscribed in a cube with its vertices at the centers of the faces of the cube. A smaller cube is then inscribed in the octahedron, with its vertices at the centroids of the faces of the octahedron. What is the ratio of the volume of the smaller cube to that of the larger?

    (A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{9}$ (E) $\frac{1}{27}$

15. Let $g(x) = 3x^3 - 2x^2 + 7x - 9$. Find

$$g(x+7) - g(x+6) - g(x+5) + g(x+4) - g(x+3) + g(x+2) + g(x+1) - g(x).$$

    (A) 144 (B) $72x + 72$ (C) 288 (D) $72x + 144$ (E) 72

1. If n is a positive integer such that $1^2 + 2^2 + 3^2 + \cdots + n^2 = 1 + 2 + 3 + \cdots + 2n$, what is n?

2. Maria and Joe are jogging towards each other on a long straight path. Joe is running at 10 mph and Maria at 8 mph. When they are 3 miles apart, a fly begins to fly back and forth between them at a constant rate of 15 mph, turning around instantaneously whenever it reaches one of the runners. How far, in miles, will the fly have traveled when Joe and Maria pass each other?

3. Find the smallest positive integer k such that $k!$ ends in at least 43 zeroes.

4. Circles centered at P and Q are externally tangent at T. Points A and C are on $\odot P$, and points B and D are on $\odot Q$ such that $\overline{AB}$ and $\overline{CD}$ are tangent to both $\odot P$ and $\odot Q$. Neither $\overline{AB}$ nor $\overline{CD}$ intersects $\overline{PQ}$. Line k is drawn through T such that k is tangent to both $\odot P$ and $\odot Q$. Line k intersects $\overline{AB}$ and $\overline{CD}$ at X and Y, respectively. If $PT = 2TQ = 12$, find XY.

5. A square sheet of paper that measures 18 cm on a side has corners labeled A, B, C, and D in clockwise order. Point B is folded over to a point E on $\overline{AD}$ with $DE = 6$ cm and the paper is creased. When the paper is unfolded, the crease intersects side $\overline{AB}$ at F. Find the number of centimeters in FB.

6. Each of the first 150 positive integers is painted on a different marble, and the 150 marbles are placed in a bag. If n marbles are chosen (without replacement) from the bag, what is the smallest value of n such that we are guaranteed to choose three marbles with consecutive numbers?

7. Points P and Q are on hypotenuse $\overline{XY}$ of $\triangle XYZ$ such that $XP = PQ = QY = XY/3$. Find all possible values of XY if $ZP^2 + ZQ^2 = 15$.

8. Find all ordered pairs (x, y) that satisfy the system of equations:

$$\begin{aligned}(x+y)^{2/3} + 2(x-y)^{2/3} &= 3(x^2-y^2)^{1/3},\\ 3x - 2y &= 13.\end{aligned}$$